DISTRIBUTION OF SEQUENCES: A THEORY

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Abstract

In this book we develop the theory of distribution of sequences which we shall identify with the set of distribution functions of sequences. [1]

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1 Preface

Let $f(x)$ be a measurable function defined on $[0, \infty)$. In the probabilistic theory $f(x)$ is called a random variable and $g(x) = |f^{-1}([0, x))|$ is a uniquely defined distribution function of $f(x)$. Here $|X|$ is the Lebesgue measure of the set X. In the uniform distribution theory a random variable is replaced by a sequence x_n, $n = 1, 2, \dots$, $x_n \in [0, 1)$. The sequence x_n can have infinitely many distribution functions defined as all possible limits

$$\frac{\#\{n \le N_k; x_n \in [0, x)\}}{N_k} \to g(x)$$

as $k \to \infty$. We shall denote by $G(x_n)$ the set of all such $g(x)$ and we shall identify with $G(x_n)$ the notion of the distribution of x_n , i.e., the distribution of x_n is known if we know the set $G(x_n)$. The importance of the set $G(x_n)$ is reflected in the fact that most properties of a sequence x_n expressed in